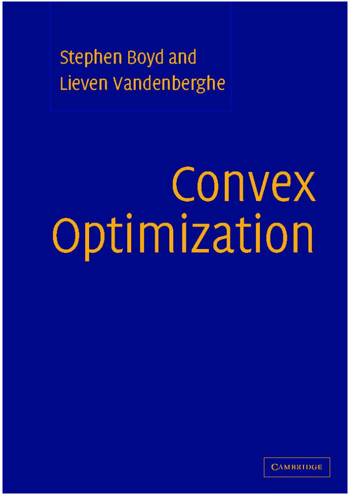

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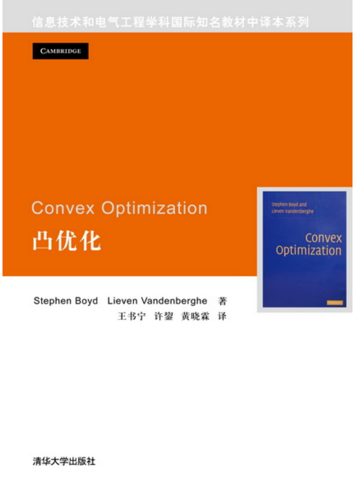

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凸优化

Stephen Boyd and
Lieven Vandenberghe
Convex
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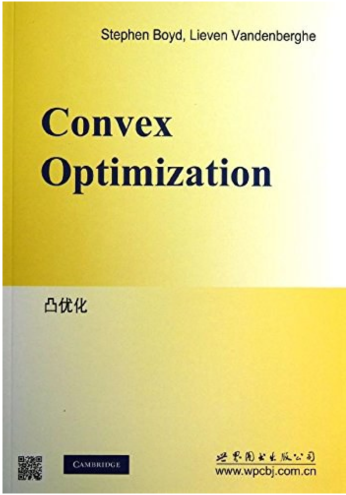
Stephen Boyd Lieven Vandenberghe 著
王书宁 许鋆 黄晓霖 译

清华大学出版社



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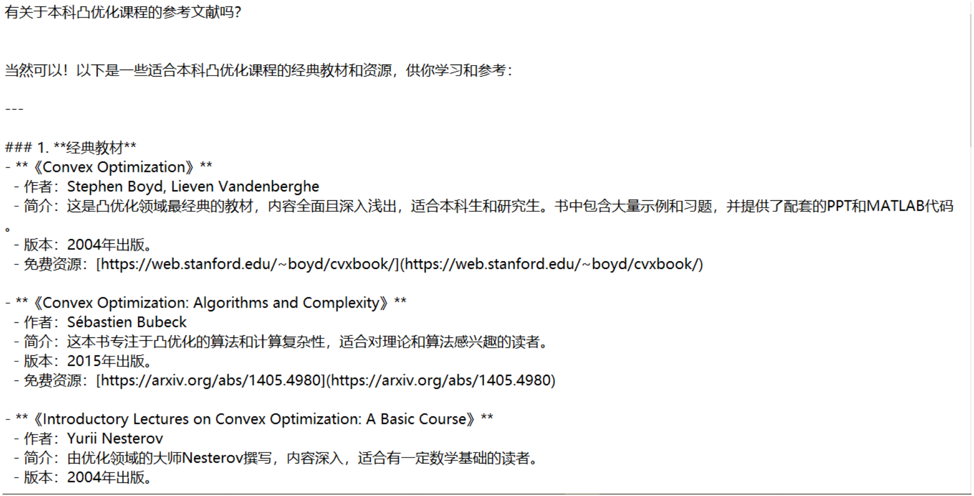
the original blue-covered version (uploaded on BB) shall prevail in case of discrepancy



有关于本科凸优化课程的参考文献吗?

当然可以！以下是一些适合本科凸优化课程的经典教材和资源，供你学习和参考：

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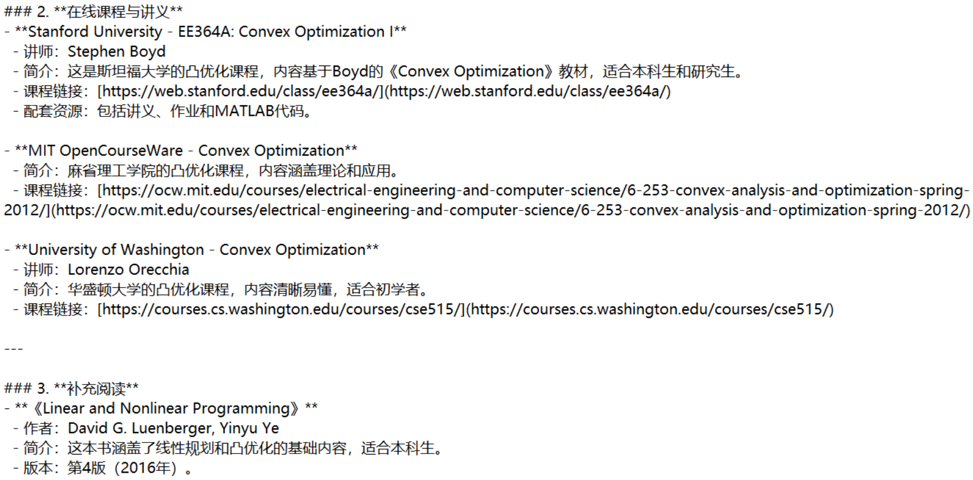
1. **经典教材**
- **《Convex Optimization》**
  - 作者：Stephen Boyd, Lieven Vandenberghe
  - 简介：这是凸优化领域最经典的教材，内容全面且深入浅出，适合本科生和研究生。书中包含大量示例和习题，并提供了配套的PPT和MATLAB代码。
  - 版本：2004年出版。
  - 免费资源：https://web.stanford.edu/~boyd/cvxbook/

- **《Convex Optimization: Algorithms and Complexity》**
  - 作者：Sébastien Bubeck
  - 简介：这本书专注于凸优化的算法和计算复杂性，适合对理论和算法感兴趣的读者。
  - 版本：2015年出版。
  - 免费资源：https://arxiv.org/abs/1405.4980

- **《Introductory Lectures on Convex Optimization: A Basic Course》**
  - 作者：Yurii Nesterov
  - 简介：由优化领域的大师Nesterov撰写，内容深入，适合有一定数学基础的读者。
  - 版本：2004年出版。

answers from DeepSeek



```
### 2. **在线课程与讲义**
- **Stanford University - EE364A: Convex Optimization I**
  - 讲师：Stephen Boyd
  - 简介：这是斯坦福大学的凸优化课程，内容基于Boyd的《Convex Optimization》教材，适合本科生和研究生。
  - 课程链接：[https://web.stanford.edu/class/ee364a/](https://web.stanford.edu/class/ee364a/)
  - 配套资源：包括讲义、作业和MATLAB代码。

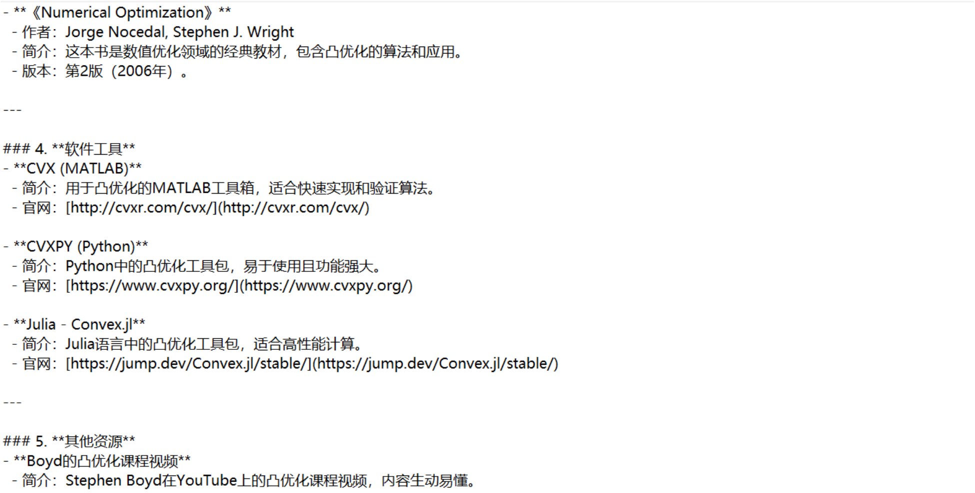
- **MIT OpenCourseWare - Convex Optimization**
  - 简介：麻省理工学院的凸优化课程，内容涵盖理论和应用。
  - 课程链接：[https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-253-convex-analysis-and-optimization-spring-2012/](https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-253-convex-analysis-and-optimization-spring-2012/)

- **University of Washington - Convex Optimization**
  - 讲师：Lorenzo Orecchia
  - 简介：华盛顿大学的凸优化课程，内容清晰易懂，适合初学者。
  - 课程链接：[https://courses.cs.washington.edu/courses/cse515/](https://courses.cs.washington.edu/courses/cse515/)

---

### 3. **补充阅读**
- **《Linear and Nonlinear Programming》**
  - 作者：David G. Luenberger, Yinyu Ye
  - 简介：这本书涵盖了线性规划和凸优化的基础内容，适合本科生。
  - 版本：第4版（2016年）。
```

answers from DeepSeek



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- ** 《Numerical Optimization》 **
  - 作者：Jorge Nocedal, Stephen J. Wright
  - 简介：这本书是数值优化领域的经典教材，包含凸优化的算法和应用。
  - 版本：第2版（2006年）。

---

### 4. **软件工具**
- **CVX (MATLAB)**
  - 简介：用于凸优化的MATLAB工具箱，适合快速实现和验证算法。
  - 官网：[http://cvxr.com/cvx/](http://cvxr.com/cvx/)

- **CVXPY (Python)**
  - 简介：Python中的凸优化工具包，易于使用且功能强大。
  - 官网：[https://www.cvxpy.org/](https://www.cvxpy.org/)

- **Julia - Convex.jl**
  - 简介：Julia语言中的凸优化工具包，适合高性能计算。
  - 官网：[https://jump.dev/Convex.jl/stable/](https://jump.dev/Convex.jl/stable/)

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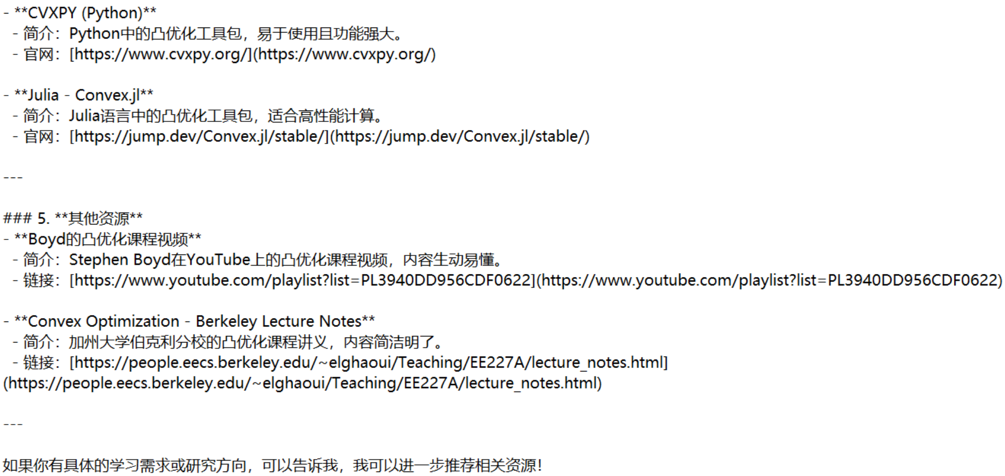
### 5. **其他资源**
- **Boyd的凸优化课程视频**
  - 简介：Stephen Boyd在YouTube上的凸优化课程视频，内容生动易懂。
```

answers from DeepSeek



- **CVXPY (Python)**
  - 简介：Python中的凸优化工具包，易于使用且功能强大。
  - 官网：https://www.cvxpy.org/

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  - 简介：Julia语言中的凸优化工具包，适合高性能计算。
  - 官网：https://jump.dev/Convex.jl/stable/

---

5. **其他资源**
- **Boyd的凸优化课程视频**
  - 简介：Stephen Boyd在YouTube上的凸优化课程视频，内容生动易懂。
  - 链接：https://www.youtube.com/playlist?list=PL3940DD956CDF0622

- **Convex Optimization - Berkeley Lecture Notes**
  - 简介：加州大学伯克利分校的凸优化课程讲义，内容简洁明了。
  - 链接：[https://people.eecs.berkeley.edu/~elghaoui/Teaching/EE227A/lecture_notes.html]
(https://people.eecs.berkeley.edu/~elghaoui/Teaching/EE227A/lecture_notes.html)

如果你有具体的学习需求或研究方向，可以告诉我，我可以进一步推荐相关资源！

answers from DeepSeek

how to achieve the most from the course

read textbook carefully	the most important ever
solve as many exercises as possible	extremely important
complete group project and exam	very important
attend lectures and participate	important

regular performance: 30%

- biweekly exercises (ample flexibility, solutions will be provided)
- possibly a total of 3 tests lasting for approximately 45 minutes each
- tentative test dates: first lecture in March, April and May

group project: 20%

- start to search for possible topics as early as possible
- tentative presentation date: May 22 (week 16)
- tentative due date for project report: May 28 (week 16)

attendance: 10%

- lecture attendance (possibly penalty for missing lectures)

final exam: 40%

tentative schedule (subject to change)

week numbers	book contents	extra activities
1	chap 1 & appendix A	
2	chap 2	
3	chap 2 & chap 3	
4	chap 3	
5	chap 4	unit test
6	chap 4	
7	chap 5	
8	chap 5 & appendix C	
9	chap 9	unit test
10	chap 9 & chap 10	
11	chap 10 & chap 11	
12	chap 11	
13	chap 6 & chap 7	
14	chap 7 & chap 8	unit test
15	chap 8 & review	
16	N/A	project presentations

Chapter 1 Introduction

Last update on 2025-03-01 11:34:59+08:00

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Mathematical optimization

(mathematical) optimization problem

$$\begin{array}{rll} \text{minimize} & f_0(x) & \\ \text{subject to} & f_i(x) \leq b_i, & i = 1, \cdots, m \\ & g_j(x) = 0, & j = 1, \cdots, p \end{array}$$

- optimization (decision) variables $x = (x_1, \cdots, x_n)$
- objective function $f_0 \colon \mathbb{R}^n \to \mathbb{R}$
- inequality constraint functions $f_i \colon \mathbb{R}^n \to \mathbb{R}, \quad i = 1, \cdots, m$
- equality constraint functions $g_j \colon \mathbb{R}^n \to \mathbb{R}, \quad j = 1, \cdots, p$

optimal solution x^*

the vector x that gives the smallest objective value among all vectors satisfying the constraints

Examples

portfolio optimization

- variables: amounts invested in different assets
- constraints: budget, max/min investment per asset, minimum return
- objective: overall risk or return variance

device sizing in electronic circuits

- variables: device widths and lengths
- constraints: manufacturing limits, timing requirements, maximum area
- objective: power consumption

data fitting

- variables: model parameters
- constraints: prior information, parameter limits
- objective: measure of misfit or prediction error

Solving optimization problems

general optimization problems

- very difficult to solve
- methods involve some compromises (e.g. no feasible solution, very long computation time, or not always finding the solution)

exceptions: certain problem classes can be solved efficiently and reliably

- least-square problems
- linear programming problems
- convex optimization problems

Mathematical optimization

Classification

Course outline

Least-squares

$$\text{minimize} \qquad \|Ax - b\|_2^2$$

solving least-squares

- analytic solution: $x^* = (A^T A)^{-1} A^T b$
- reliable and efficient algorithms and software
- computation time proportional to $n^2 k$ (when $A \in \mathbb{R}^{k \times n}$); less if structured
- a mature technology

using least-squares

- least-squares problems are easy to recognize
- a few standard techniques increase flexibility (e.g. including weights, adding regularization terms)

Linear programming

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & a_i^T x \leq b_i, \qquad i = 1, \cdots, m \end{array}$$

solving linear programs

- no analytical formula for solution
- reliable and efficient algorithms and software
- computation time proportional to $n^2 m$ if $m \geq n$; less if structured
- a mature technology

using linear programming

- no as easy to recognize as least-squares problems
- a few standard tricks used to convert problems into linear programs (e.g. problems involving ℓ_1- or ℓ_∞-norms, piecewise linear functions)

Convex optimization problems

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \leq b_i, \qquad i = 1, \cdots, m \end{array}$$

- objective and constraint functions are convex

  $$f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y)$$

  if $\alpha, \beta \geq 0$ and $\alpha + \beta = 1$
- includes least-squares problems and linear programs as special cases

solving convex optimization problems

- no analytical solution
- reliable and efficient algorithms
- computation time (roughly) proportional to $\mathbf{max}\{n^3, n^2m, F\}$ where F is the cost of evaluating f_i's and their first and second derivatives
- almost a technology

using convex optimization

- often difficult to recognize
- many tricks for transforming problems into convex form
- surprisingly many problems can be solved via convex optimization

Nonlinear (nonconvex) optimization

traditional techniques for general nonconvex problems involve compromises

local optimization methods

- find a point that minimizes the objective function among feasible points near it
- fast, can handle large problems
- require initial guess
- provide no information about distance to global optimum

global optimization methods

- find the global solution
- worst-case complexity grows exponentially with problem size

the above algorithms are often based on solving convex subproblems

roles of convex optimization in nonconvex problems

- initialization for local optimization
- convex heuristics for nonconvex optimization
- bounds for global optimization

Mathematical optimization

Classification

Course outline

Course outline

theory

- basic convex analysis
- recognize and formulate problems as convex optimization problems
- Lagrangian duality, characterize optimal solutions

algorithms

- problem types: unconstrained, equality constrained, inequality constrained
- algorithms: Newton's algorithm, interior-point methods

applications

- data fitting, probability and statistics, computational geometry

Modeling languages for convex optimization

- domain specific languages (DSLs) for convex optimization
  - describe problem in high level language, close to the math
  - can automatically transform problem to standard form, then solve
- enables rapid prototyping
- it's now much easier to develop an optimization-based application
- ideal for teaching and research (can do a lot with short scripts)

- gets close to the basic idea: say what you want, not how to get it

CVXPY example: non-negative least squares

math:

$$\begin{array}{ll} \text{minimize} & \|Ax - b\|_2^2 \\ \text{subject to} & x \geq 0 \end{array}$$

- variable is x
- A, b given
- $x \geq 0$ means $x_1 \geq 0, \ldots, x_n \geq 0$

CVXPY code:

```
import cvxpy as cp

A, b = ...

x = cp.Variable(n)
obj = cp.norm2(A @ x - b)**2
constr = [x >= 0]
prob = cp.Problem(cp.Minimize(obj), constr)
prob.solve()
```

Brief history of convex optimization

theory (convex analysis): 1900-1970

algorithms

- 1947: simplex algorithm for linear programming (Dantzig)
- 1960s: early interior-point methods (Fiacco & McCormick, Dikin, . . .)
- 1970s: ellipsoid method and other subgradient methods
- 1984: polynomial-time interior-point methods for linear programming (Karmarkar)
- 1994: polynomial-time interior-point methods for nonlinear convex optimization (Nesterov & Nemirovski)

applications

- before 1990: mostly in operations research; few in engineering
- since 1990: many new applications in engineering (control, signal processing, communications, circuit design, . . .); new problem classes (semidefinite and second-order cone programming, robust optimization, . . .)
- since 2000s: machine learning and statistics, finance

Summary

convex optimization problems

- are optimization problems of a special form
- arise in many applications
- can be solved effectively
- are easy to specify using DSLs